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GREEN'S FUNCTION METHOD IN THE INVESTIGATION OF DYNAMIC STABILITY OF A FLUID-CONVEYING PIPE

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ABSTRACT

Fluid-conveying pipes represent a fundamental dynamic problem within the realm of fluid-structure interaction. They find extensive applications in various industries, including petroleum, nuclear engineering, aviation, aerospace, and nanostructures. This paper applies the Green's function method to solve the stability problem of a fluid-conveying pipe, hinged at both ends and supported by intermediate linear-elastic supports. The objective is to examine the influence of the number and rigidity of these supports on the critical fluid velocity, which is the velocity at which the pipe loses stability. A numerical solution was performed for a straight pipe conveying fluid with specified geometric and physical characteristics, where the number and rigidity of the elastic supports were considered as parameters. The numerical analysis presented herein includes graphs illustrating the dependence of the critical fluid velocity on the number of elastic supports for varying support rigidities. These results reveal that the elastic supports affect both the vibrational characteristics and the critical velocity of the conveyed fluid. The solution results are compared with those obtained using one of the most widely employed methods for analyzing the dynamic stability of pipe systems (Transfer Matrix Method – TMM). A good agreement between the results is observed. The paper aims at presenting a method for obtaining the exact solution to the differential equation governing the lateral displacements of a pipe system. This paper discusses the authors' perceived pros and cons of the Green's function method in comparison to the most popular methods for the dynamic investigation of fluid-conveying pipes.

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МЕТОД ФУНКЦИИ ГРИНА В ИССЛЕДОВАНИИ ДИНАМИЧЕСКОЙ УСТОЙЧИВОСТИ ТРУБОПРОВОДА, ТРАНСПОРТИРУЮЩЕГО ЖИДКОСТЬ

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АННОТАЦИЯ

Трубы для транспортировки жидкости представляют собой фундаментальную динамическую проблему в области взаимодействия жидкости и конструкции. Они находят широкое применение в различных отраслях промышленности, включая нефтяную, ядерную, авиационную, аэрокосмическую и наноструктурную. В данном исследовании метод функции Грина применяется для решения проблемы устойчивости трубы для транспортировки жидкости, шарнирно закрепленной на обоих концах и поддерживаемой промежуточными линейно-упругими опорами. Цель состоит в том, чтобы изучить влияние количества и жесткости этих опор на критическую скорость жидкости, являющиеся скоростью, при которой труба теряет устойчивость. Численное решение было выполнено для прямой трубы, транспортирующей жидкость с заданными геометрическими и физическими характеристиками, где количество и жесткость упругих опор рассматривались как параметры. Представленный здесь численный анализ включает графики, иллюстрирующие зависимость критической скорости жидкости от количества упругих опор для различной жесткости опор. Эти результаты показывают, что упругие опоры влияют как на вибрационные характеристики, так и на критическую скорость транспортируемой жидкости. Результаты решения сравниваются с данными, полученными с помощью одного из наиболее широко используемых методов анализа динамической устойчивости трубопроводных систем (метод матрицы переноса – ТММ). Наблюдается хорошее соответствие между результатами. Целью статьи является представление метода получения точного решения дифференциального уравнения, регулирующего боковые смещения трубопроводной системы. Обсуждаются предполагаемые авторами плюсы и минусы метода функции Грина в сравнении с наиболее популярными методами динамического исследования трубопроводов, транспортирующих жидкости.

Introduction

Fluid conveying pipes represent a fundamental challenge in the field of fluid-structure interaction. The issue of dynamic stability of such structures has garnered significant attention in both scientific research and industry. Given that fluid-flowing pipes are integral components in numerous engineering facilities and serve as a primary means of transporting oil and gas, the stability of these pipes is of paramount importance. The loss of stability in a pipe can result in damages that have far-reaching consequences on the economy, the environment, and the well-being of the population.

M. P. Paidoussis is a distinguished scientist renowned for numerous publications in the field of fluid-structure interaction. His research, particularly focused on the interaction between flowing fluids and pipes, is extensively discussed in [1] and [2].

In their works [3] and [4], R. Gregory and M. P. Paidoussis present numerical studies and results from experiments conducted on the dynamic stability of cantilevered pipes with flowing fluid.

In [5], an approach is presented for determining the circular frequencies and oscillations of a pipe conveying fluid. The transverse displacements of the pipe axis are also calculated. Additionally, [6] explores the vibrations of a pipe with a transverse linear elastic support.

A recent study on fluid-conveying pipes was conducted in [44]. Using the Euler-Bernoulli model, the authors investigated the stability of a pipe resting on a two-parameter elastic foundation, considering various boundary conditions. The Differential Transform Method was employed in the analysis. The results indicate that the elastic foundation has a stabilizing effect on the system, and increasing the foundation parameters leads to an increase in the system's critical velocity.

In a further recent investigation [45], the dynamic stability of a cantilevered pipe subjected to a lateral distributed load was analyzed. The Differential Quadrature Method was utilized in the analysis.

Ding and Ji [46] present a comprehensive review of the latest research in the field of vibration control for fluid-conveying pipes.

The broad applicability of nanoscale tubes across various scientific and industrial domains has sparked considerable research, as demonstrated in [9–16] and more recently in [47–49]. To overcome the complexities and expenses of nanoscale experiments, fluid-structure interaction in carbon nanotubes is frequently studied using continuum elastic models, such as Euler and Timoshenko beam theories.

Curved pipes represent another area of fluid-structure interaction. Research in this area has been conducted in [33–38].

Other aspects of the problem of fluid-conveying pipe dynamic stability include pipes under thermal loads [26; 27], submerged pipe systems [19–21], and pipes made of viscoelastic materials [23].

Common methods for dynamic analysis of fluid-conveying pipes include the Finite Element Method (FEM), the Transfer Matrix Method (TMM) [39], and the Generalized

Differential Quadrature Method (GDQM). However, this paper proposes a different approach by suggesting the use of the Green's Function Method to solve the stability problem. The objective is to demonstrate that this method can be competitive with the well-established techniques mentioned earlier in the dynamic stability investigations of fluid-conveying pipes.

The study in this paper focuses on a fluid-conveying pipe supported by linear elastic springs. The results obtained shed light on the relationship between the critical fluid velocity and the rigidity, as well as the number of elastic supports. The critical fluid velocity is the speed at which the flowing fluid leads to the loss of stability in the pipe.

The paper is structured as follows: first, we present the model of the pipe, including its static scheme, and the governing differential equation for eigen lateral vibrations. Second, we employ the Green's function method to solve the problem, demonstrating the derivation of the frequency equation of the system. Conclusions about the system's stability can be drawn based on the roots of this equation. Finally, we present the results obtained from the numerical solution and summarize several key conclusions. To verify the results, they are compared to numerical results obtained using the Transverse Matrix Method for the same system. Furthermore, we **offer** our perspective on the key advantages and disadvantages of the Green's function method compared to other methods for the dynamic investigation of pipe systems.

Problem formulation

The pipe depicted in figure 1 is presented for analysis. It is hinged at it's both ends and on intermediate linear elastic supports. The differential equation below describes the lateral vibration of the pipe.

$$EI\frac{\partial^4 w}{\partial x^4}+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}+2m_f V\frac{\partial^2 w}{\partial t\,\partial x}+ \\ +m_f V^2\frac{\partial^2 w}{\partial x^2}+w\sum_{j=1}^{n}K_j\delta\left(x-x_j\right)=0. \quad (1)$$

In equation (1), t is the time and x is the axial coordinate of a random cross-section of the pipe (Fig.1). $w\left(x,t\right)$ denotes the function describing the transverse displacements of the pipe's axis. The remaining system characteristics include the velocity of the conveyed fluid V, the bending stiffness of the pipe EI, the mass of the fluid per unit length of the pipe m_f, the mass of the pipe per unit length m_p and the rigidity of the linear intermediate supports K_j. δ is the Dirac delta function.

Equation (1) is valid under the following assumptions [1; 2; 8]:

a) Dissipation and damping effects are neglected;

b) Transported fluid is heavy, unviscous, and incompressible. This allows us to assume that the fluid velocity is uniform across the cross-section;

c) The Euler-Bernoulli hypothesis is valid. This implies that plane cross sections, orthogonal to the axis of the pipe before deformation, remain plane and orthogonal to the deformed axis of the pipe;

d) Length of the pipe is significantly greater than the dimensions of the cross section;

e) The pipe is assumed axially inextensible;

f) The effects of rotary inertia are neglected;

g) The material of the pipe is linear elastic (The Hooke's law is valid);

Each term in equation (1) has the following physical interpretation:

a) Coriolis force:

$$2m_f V\frac{\partial^2 w}{\partial t\,\partial x}.$$

b) Inertial force:

$$\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}.$$

c) Centripetal force (arising due to the acceleration of fluid through the deformed pipe):

$$m_f V^2\frac{\partial^2 w}{\partial x^2}.$$

a) The force in the *j*-th elastic support

$$w\sum_{j=1}^{n}K_j\delta\left(x-x_j\right)=0.$$

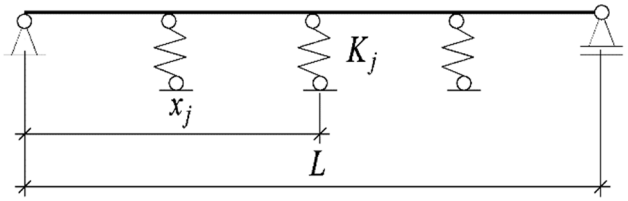


Fig. 1. Static scheme of the pipe

The dimensional equation (1) is converted into a non-dimensional form by introducing the following dimensionless parameters.

$$\xi=\frac{x}{L};\quad \xi_j=\frac{x_j}{L};\quad y=\frac{w}{L};\quad u=VL\sqrt{\frac{m_f}{EI}};\quad \beta=\frac{m_f}{m_f+m_p};$$

$$\tau=\frac{t}{L^2}\sqrt{\frac{EI}{m_f+m_p}};\quad \Omega=\omega L^2\sqrt{\frac{EI}{m_f+m_p}};\quad k_j=\frac{K_j L^3}{EI}, \quad (2)$$

where β is the mass ratio, u represents the dimensionless parameter associated with the fluid velocity, τ is the dimensionless parameter associated with the time, Ω is the non-dimensional parameter of the pipe's circular frequency and k_j is the non-dimensional parameter of the rigidity of the elastic supports.

Following the mentioned transformation, equation (1) takes the following form:

$$\frac{\partial^4 y}{\partial \xi^4}+u^2\frac{\partial^2 y}{\partial \xi^2}+2\sqrt{\beta}u\frac{\partial^2 y}{\partial \tau \partial \xi}+\frac{\partial^2 y}{\partial \tau^2}+y\sum_{j=1}^{n}k_j\,\delta\left(\xi-\xi_j\right)=0. \quad (3)$$

The solution of the differential equation (3) is sought in the form:

$$y(\xi,\tau)=W(\xi)e^{i\Omega\tau}, \quad (4)$$

where i is the imaginary unit.

By substituting (4) in (3) the following equation is derived:

$$W^{IV}+u^2W^{II}+2\sqrt{\beta}ui\,\Omega W^{I}-\Omega^2 W+W\sum_{j=1}^{n}k_j\,\delta\left(\xi-\xi_j\right)=0. \quad (5)$$

The subsequent property of the Dirac delta function is employed [7]:

$$y(\xi)\delta\left(\xi-\xi_j\right)=y\left(\xi_j\right)\delta\left(\xi-\xi_j\right). \quad (6)$$

As a result, equation (5) obtains the following form:

$$W^{IV}+u^2W^{II}+2\sqrt{\beta}ui\,\Omega W^{I}-\Omega^2 W=-\sum_{j=1}^{n}k_j W\left(\xi_j\right)\delta\left(\xi-\xi_j\right) \quad (7)$$

The solution of equation (7) can be expressed in this manner:

$$W(\xi)=-\sum_{j=1}^{n}k_j W\left(\xi_j\right)G\left(\xi,\xi_j\right), \quad (8)$$

where $G\left(\xi;\xi_j\right)$ is the Green function, which satisfies the equation:

$$\frac{\partial^4 G}{\partial \xi^4}+u^2\frac{\partial^2 G}{\partial \xi^2}+2\sqrt{\beta}ui\,\Omega\frac{\partial G}{\partial \xi}-\Omega^2 G=\delta\left(\xi-\xi_j\right). \quad (9)$$

The Green's function satisfies the same boundary conditions as $W(\xi)$. For the pipe in Fig. 1, these conditions are:

$$G\left(0;\xi_j\right)=G\left(1;\xi_j\right)=0;$$

$$G^{II}\left(0;\xi_j\right)=G^{II}\left(1;\xi_j\right)=0. \quad (10)$$

In equation (8) is substituted $\xi=\xi_i$, $i=1,...,n$, resulting in the following system of equations

$$W\left(\xi_j\right)+\sum_{j=1}^{n}k_j W\left(\xi_j\right)G_{ij}=0. \quad (11)$$

The system (11) has a nontrivial solution if:

$$\begin{vmatrix} k_1G_{11}+1 & k_2G_{12} & \cdots & k_nG_{1n} \\ k_1G_{21} & k_2G_{22}+1 & \cdots & k_nG_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_1G_{n1} & k_2G_{n2} & \cdots & k_nG_{nn}+1 \end{vmatrix}=0. \quad (12)$$

Equation (12) is the frequency equation of the system. From it, the circular frequencies $\omega_1,...,\omega_n$ could be obtained.

The solution of equation (9) is as follows [8]:

$$\begin{aligned} G\left(\xi;\xi_j\right)=H\left(\xi-\xi_j\right)\sum_{\alpha_i}\frac{e^{\alpha_i\left(\xi-\xi_j\right)}}{4\alpha_i^3+2\alpha_i a+ib}+ \\ +W(0)\sum_{\alpha_i}\frac{\alpha_i^3 e^{\alpha_i\xi}}{4\alpha_i^3+2\alpha_i a+ib}+ \\ +W^{I}(0)\sum_{\alpha_i}\frac{\alpha_i^2 e^{\alpha_i\xi}}{4\alpha_i^3+2\alpha_i a+ib}+ \\ +\left[W^{III}(0)+aW^{I}(0)+biW(0)\right]\sum_{\alpha_i}\frac{e^{\alpha_i\xi}}{4\alpha_i^3+2\alpha_i a+ib}+ \\ +\left[W^{II}(0)+aW(0)\right]\sum_{\alpha_i}\frac{\alpha_i e^{\alpha_i\xi}}{4\alpha_i^3+2\alpha_i a+ib}. \end{aligned} \quad (13)$$

In equation (13) $a=u^2$, $b=2\sqrt{\beta}u\Omega$ and $c=\Omega^2$. $H\left(\xi;\xi_j\right)$ is the Heaviside function and the coefficients $\alpha_i, i=1,....,4$ are obtained as roots of the equation [8]:

$$z^4+a\,z^2+i\,b\,z-c=0. \quad (14)$$

The subsequent function is defined

$$\psi(\xi)=\sum_{\alpha_i}\frac{e^{\alpha_i\xi}}{4\alpha_i^3+2\alpha_i a+ib}. \quad (15)$$

Then, equation (15), for the pipe in figure 1, takes the form

$$\begin{aligned} G\left(\xi;\xi_j\right)=H\left(\xi-\xi_j\right)\psi\left(\xi-\xi_j\right)+ \\ +\psi^{II}(\xi)W^{I}(0)+\psi(\xi)\left[W^{III}(0)+aW^{I}(0)\right]. \end{aligned} \quad (16)$$

The values for $W^{I}(0)$ and $W^{III}(0)$ are obtained from the boundary conditions at the right end of the pipe.

$$W(1)=0 \;\rightarrow\; G\left(1;\xi_j\right)=0; \quad (17)$$

$$W^{II}(1)=0 \;\rightarrow\; G^{II}\left(1;\xi_j\right)=0. \quad (18)$$

Equations (17) and (18) are transformed in the form:

$$\begin{aligned} 0=\psi\left(1-\xi_j\right)+\psi^{II}(1)W^{I}(0)+ \\ +\psi(1)\left[W^{III}(0)+aW^{I}(0)\right]; \end{aligned} \quad (19)$$

$$\begin{aligned} 0=\psi^{II}\left(1-\xi_j\right)+\psi^{IV}(1)W^{I}(0)+ \\ +\psi^{III}(1)\left[W^{III}(0)+aW^{I}(0)\right]. \end{aligned} \quad (20)$$

To obtain the critical velocity of the flowing fluid V_{cr}, an iteration procedure is applied. The velocity of the fluid is varied from 0 to the critical value V_{cr}, which corresponds to the zero value of the system's first natural frequency.

Numerical results

Numerical investigations were conducted for the fluid-conveying pipe in Figure 1. Only the case with equal distances between the supports of the pipe has been considered. The material and geometric properties of the system are: modulus of elasticity of the material of the pipe $E = 210\,GPa$, density of the pipe's material $\rho = 7800\,kg/m^3$. The outer and the inner radii of the pipe's cross-section are $R_{out} = 0.014\,m$ and $R_{in} = 0.012\,m$. Three different fluids with the following densities were considered: $\rho = 1000\,kg/m^3$; $\rho = 1100\,kg/m^3$ and $\rho = 1200\,kg/m^3$.

Figure 2 depicts the dependence of the critical velocity on the number of intermediate supports and their rigidity for a fluid with density $\rho = 1000\,kg/m^3$.

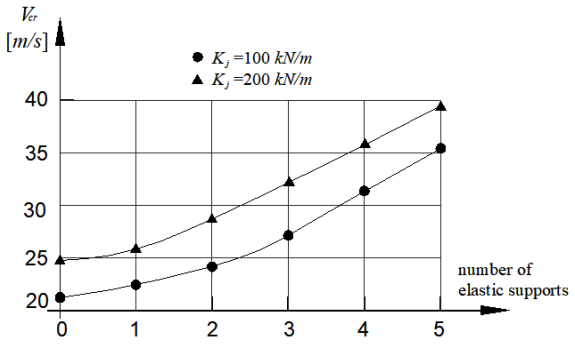


Fig. 2. Critical fluid velocity versus the number of intermediate supports for a fluid with density $\rho = 1000\,kg/m^3$

Figures 3 and 4 show the dependence of the critical fluid velocity on the number of intermediate supports for different fluid densities.

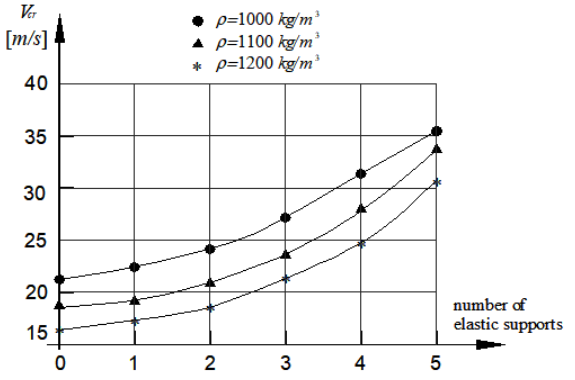


Fig. 3. Critical fluid velocity versus the number of intermediate supports for three different fluids with rigidity of the elastic supports $K_j = 100\,kN/m$

A comparison has been made between the results obtained using the Green's function method and the solution for the same system using the Transfer Matrix Method (TMM). When 10 eigenforms are used in the TMM solution, the results show an average difference of 5.4 % in the critical velocities obtained. As expected, increasing the number of eigenforms to 15 reduces the difference in the results of both methods to 3.3 %.

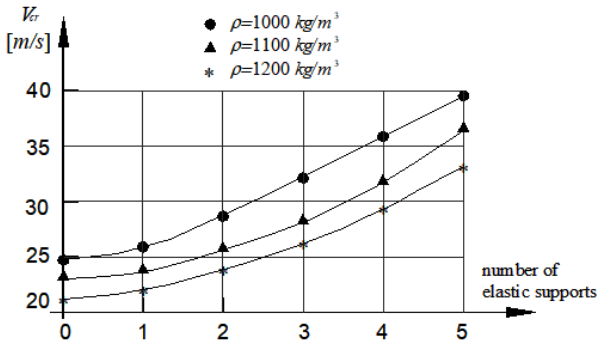


Fig. 4. Critical fluid velocity versus the number of intermediate supports for three different fluids with rigidity of the elastic supports $K_j = 200\,kN/m$

Conclusion

The utilization of the Green's function method in this paper offers an alternative avenue for exploring the dynamic response of fluid-conveying pipes. The suggested approach can be regarded as a viable competitor to other established methods for dynamic investigation of fluid-conveying pipes, such as the Transferred Matrix Method, Generalized Differential Quadrature Method (GDQM) and Finite Element Method (FEM).

In the scientific literature, it has been shown that TTM and GDQM have a significant advantage over FEM. The FEM's application to pipelines with numerous spans results in a substantial increase in computational time, attributed to the expanding order of the system's property matrices. This contrasts sharply with the TMM, where the overall transfer matrix retains a fixed order, independent of the number of spans. The GDQM efficiently approximates derivatives in the pipe's lateral vibration equation by utilizing weighted sums of function values at discrete points, resulting in rapid convergence with a sparse grid.

Compared to the TMM, the Green's function method has the advantage of not requiring the solution of the free vibration problem to determine eigenvalues and eigenfunctions, which are also essential in the classical Galerkin method.

Another significant advantage of the Green's function method is that the accuracy of the solution does not depend on the number of finite elements (as in FEM) or the number of eigenforms used in the Galerkin method and TMM, as the method is regarded as equivalent to the exact analytical solution of equation (1) [8].

The suggested approach also has a significant drawback: it requires the use of a complex mathematical framework, which could make it less appealing for practical engineering applications.

Several major conclusions can be drawn regarding the numerical investigations presented in the article:

1. The numerical results indicate that augmenting the number of intermediate supports enhances the system stability.

2. Another notable observation is that boosting the rigidity of the elastic supports results in increased system stability.

3. The findings demonstrate that the dynamic response of the system is notably affected by the fluid velocity, which could potentially jeopardize its safety.

4. Increasing the density of the fluid leads to a decrease in the stability of the system.

References

1. Paidoussis M. Fluid-structure interactions: Volume 1 Slender Structures and Axial flow. Academic Press, London, 2013.

2. Paidoussis M. Fluid-structure interactions: Volume 2 Slender Structures and Axial flow. Academic Press, London, 2016.

3. Gregory R., Paidoussis M. Unstable oscillation of tubular cantilevers conveying fluid I. Theory. *Proceedings of the Royal Society of London.* 1966, Vol. 293, No 1435, pp. 512-527.

4. Gregory R., Paidoussis M. Unstable oscillation of tubular cantilevers conveying fluid II. Experiments. *Proceedings of the Royal Society of London*. 1966, Vol. 293, No 1435, pp. 528-542.

5. Udoetok E. Internal Fluid Flow Induced Vibration of Pipes. *Journal of Mechanical Design and Vibration*. 2018, Vol. 6, No. 1, pp. 1–8.

6. Faal R.T., Derakhshan D. Flow-Induced Vibration of Pipeline on Elastic Support. *Procedia Engineering*. 2011, Vol. 14, pp. 2986-2993.

7. Wheeler N. Dirac delta function identities, Reed College, Physics Department, USA, 1997.

8. Tembely M. Analytical solution of a forced vibrating tube transporting fluids. *preprints.org,* (*in press*). 2024.

9. Reddy, C.D., Lu C., Rajendran S., Liew K.M. Free vibration analysis of fluid conveying single-walled carbon nanotubes. *Applied Physics Letters.* 2007, Vol. 90, pp. 133122.

10. Yoon, J., Ru C., Mioduchowski A. Vibration and instability of carbon nanotubes conveying fluid. *Composites Science and Technology*. 2005, Vol.65 - pp. 1326 - 1336.

11. Lolov D.S., Lilkova-Markova Sv. V. Dynamic stability of double-walled carbon nanotubes. *Journal of the Serbian Society for Computational Mechanics.* 2018, Vol.12(1), pp.1-8.

12. Yoon J., Ru C.Q., Mioduchowski A, Flow-induced flutter instability of cantilever CNTs. *Int. J. Solids Struct*. 2006, Vol.43, pp. 3337–3349.

13. Yoon J., Ru C.Q., Mioduchowski A, Vibration and instability of CNTs conveying fluid. *Compos. Sci. Technol*. 2005, Vol. 65, pp. 1326–1336.

14. Wang X.Y., Wang X., Sheng G.G. The coupling vibration of fluid-filled carbon nanotubes. *J. Phys. D: Appl. Phys*. 2007, Vol.40, pp.2563–2572.

15. Dong K., Wang X. Wave propagation in carbon nanotubes under shear deformation. *Nanotechnology.* 2006, Vol. 17, pp. 2773–2782.

16. Dong K., Wang X., Sheng G.G. Wave dispersion characteristics in fluid-filled carbon nanotubes embedded in an elastic medium. *Model. Simul. Mater. Sci. Eng*. 2007, Vol.15, pp. 427–439.

17. Jeong W., Soo Y., Jeong S.,Lee S., Yoo W. Stability Analysis of a pipe conveying periodically pulsating fluid using finite element method. *JSME International Journal*. 2006, Vol. 49, pp. 1116-1122.

18. Paidoussis M., Issid N. Dynamic stability of pipes conveying fluid. *Journal of Sound and Vibration*. 1974, Vol. 33(3), pp. 267-284.

19. Deng Q., Yang Z. Wave propagation in submerged pipe conveying fluid. Acta Mechanica Solida Sinica. 2019, Vol. 32(4), pp. 483-498.

20. Lolov D. S., Lilkova-Markova Sv. Dynamic stability of a fluid-immersed pipe conveying fluid and resting on a damped Winkler elastic foundation. *Proceedings of XI International Conference Industrial Engineering and Environment Protection 2021 (IIZS 2021)*, October 7-8th, 2021, Zrenjanin, Serbia. pp.49-55.

21. Hellum A., Mukherjee R., Hull A. Flutter instability of a fluid-conveying fluid-immerced pipe affixed to a rigid body. Journal of Fluids and Structures. 2011, Vol.27, pp. 1086-1096.

22. Lin W., Qiao N. Vibration and stability of an axially moving beam immersed in fluid. *International Journal of Solids and Structures*. 2008, Vol. 45, pp. 1445-1457.

23. Huang Q., Lin T., Safarpour M. Flow-induced vibration attenuation of a viscoelastic pipe conveying fluid under sinusoidal flow, using a nonlinear absorber. *Mechanics Based design of Structures and Machines*. 2020, Vol. 50, pp. 1673-1703.

24. Liang F., Gao A., Yang X.D. Dynamical analysis of spinning functionally graded pipes conveying fluid with multiple spans. *Applied Mathematical Modeling*. 2020, Vol. 83, pp. 454-469.

25. Askarian A.R., Permoon M.R., Shakouri M. Vibration analysis of pipes conveying fluid resting on a fractional Kelvin-Voigt viscoelastic foundation with general boundary conditions. *International Journal of Mechanical Sciences*. 2020. Vol. 179, pp. 105702.

26. Cao J., Liu Y., Liu W. The effect of two cases of temperature distributions on vibration of fluid-conveying functionally graded thin-walled pipes. *Journal of Strain Analysis*. 2018, Vol. 53(5), pp. 324-331.

27. Ameen K.A., Al-Dulaimi M.J., Hatem A.A. Experimental study of vibration on pipe conveying fluid at different end conditions for different fluid temperatures. *Engineering and Technology Journal*. 2019, Vol. 37, pp. 512-515.

28. Li B., Wang Z., Jing L. Dynamic response of pipe conveying fluid with lateral moving supports. *Shock and Vibration*. 2018, Vol. 2018, pp. 1-17.

29. Al-Waily M., Al-Baghdadi M., Al-Khayat R. Flow Velocity and Crack Angle Effect on Vibration and Flow Characterization for Pipe Induced Vibration. *International Journal of Mechanical & Mechatronics Engineering*. 2017, Vol. 17(5), pp. 19-27.

30. Siba M., Wahmahmood W., Zakinuaw M., Rasani R., Nassir M. Flow-induced vibrations in pipes: challengess and solutions – a review. *Journal of Engineering Science and Technology*. 2016, Vol. 11(3), pp. 362-382.

31. Faal R., Derakhshan D. Flow-Induced Vibration of Pipeline on Elastic Support. *Procedia Engineering*. 2022, Vol. 14, pp. 2986-2993.

32. Shiwen L., Ronghui H., Zhao Y., Jiarui W., Shuang C., Jianyong L., Yi L. Numerical Simulation Research on Flow-Induced Vibration Characteristics of Fluid-Conveying Pipe Network. *Nuclear Power Engineering*. 2022, Vol. 43(1), pp. 187-191.

33. Jung D, Chung J. In-plane and out-of-plane motions of an extensible semi- circular pipe conveying fluid. *Journal of Sound and Vibration*, 2008, Vol.311(1), pp.408–420.

34. Liu G., Li S., Karney B.Y. Vibration analysis of curved pipes conveying fluid. *Proceedings of the ASME 2014 Pressure Vessels & Piping Conference, Anaheim, California, USA*. 2014, Vol.3.

35. Lolov D., Lilkova-Markova Sv. Free vibrations out-of-plane of curved planar pipe conveying fluid in two cases of supporting. *Annual of UACEG, Mathematics. Mechanics*. 2006, Vol. 72(2), pp.123-128.

36. Liang F., Yang X.-D., Bao R.-D., Zhang W. Frequency Analysis of Functionally Graded Curved Pipes Conveying Fluid. *Advances in Materials Science and Engineering*. 2016, Vol.2016, Article ID 7574216.

37. Zhao Q., Sun Z. Flow-induced vibration of curved pipe conveying fluid by a new transfer matrix method. *Engineering applications of computational applied mechanics*. 2018, Vol.12(1), pp. 780-790.

38. Zhao Q., Sun Z. In-plane forced vibration of curved pipe conveying fluid by Green function method. *Applied Mathematics and Mechanics (English Edition)*. 2017, Vol.38(10), pp. 1397-1414.

39. Wu J., Shih P. The dynamic analysis of a multispan fluid-conveying pipe subjected to external load. *Journal of sound and vibration*. 2001, Vol.239(2), pp. 201-215.

40. Inspeger T., Horvath R. Pendulum with harmonic variation of the suspension point. *Periodica Polytechnica*. 2000, Vol. 44, pp. 39-46.

41. Akulenko L.D., Georgievskii D.V., Nesterov S.V. Spectrum of Transverse Vibrations of a Pipeline Element under Longitudinal Load. *Doklady Physics*. 2016, Vol. 61 (3), pp. 129–132.

42. Gay-Balmaz F., Georgievskii D., Putkaradze V. Stability of Helical Tubes Conveying Fluid. *Journal of Fluids and Structtures.* 2018. Vol 78 (2), pp. 146–174.

43. Il'ichev A.T. Dynamics and spectral stability of soliton-like structures in fluid-filled membrane tubes. *Russian Math. Surveys*. 2020. Vol 75 (5), pp. 843–882

44. Balkaya M., Kaya M. Analysis of the instability of pipes conveying fluid resting on two-parameter elastic soil under different boundary conditions. *Ocean Engineering*. 2021. Vol.241, pp.110003

45. Zhang Yu, Pengzhou Li. Dynamical stability of pipe conveying fluid with various lateral distributed loads. *Archive of Applied Mechanics*. 2023. Vol.93, pp. 4093-4106

46. Ding Hu, Ji J. Vibration control of fluid-conveying pipes: A state of the art review. *Appl. Math. Mech.-Engl*. 2023. Vol. 44, pp. 1423-1456

47. Xueping C., Xiaoxiang H., Congjia Q., Yinghui L. Stability and nonlinear vibration of carbon nanotubes-reinforced composite pipes conveying fluid. *Ocean Engineering*. 2023. Vol.281, pp.114960

48. Vassilev V., Valchev G. Dynamics and stability of Double-Walled Carbon Nanotube cantilevers conveying fluid in an elastic medium. *Dynamics*. 2024. Vol. 4(2), pp. 222-232

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